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# A Method for Determining the Point of Lift-Off and Modified Trajectory of a Ground-Based Heated Turbulent Planar Jet in a Co-Flowing Wind

MILTON M. KLEIN

2 February 1977



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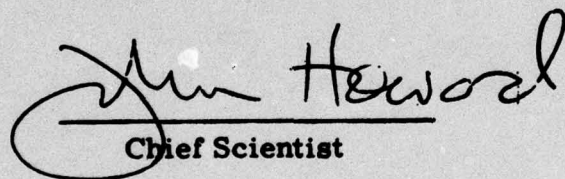


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20. Abstract (Continued)

→ To take account of the ground effect, an analysis has been made of the experimental data for the planar jet at the point of lift-off in terms of the local Froude number at this point. From this correlation a procedure has been developed for determining the lift-off point, using the ambient wind and initial velocity and temperature of the jet as input variables. A new jet trajectory may now be easily calculated with only a simple modification of the original method in which the ground effect was ignored. ←

The agreement between the calculated and experimental trajectories, obtained by the present method, is considerably improved over that yielded by the original procedure. The method of calculating the jet trajectories, when the ground effect is taken into account, appears accurate enough to be utilized in the development of a control model for the Warm Fog Dispersal System.

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## Preface

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## Contents

1. INTRODUCTION	7
2. ANALYSIS	8
3. RESULTS AND DISCUSSION	15
4. PROCEDURE FOR RAPID DETERMINATION OF LIFT-OFF POINT	19
5. SUMMARY AND CONCLUSIONS	21
REFERENCES	23

## Illustrations

1. Schematic Representation of Jet and Lift-Off Point	9
2. Plot of Lift-Off Point $x_c$ Against Parameter $G_1$ for Co-Flowing Test Data	11
3. Plot of Lift-Off Point $x_c$ Against Parameter $G$ for Co-Flowing Test Data	11
4. Plot of Parameter $p$ Against Parameter $q$ for Co-Flowing Test Data	13
5. Comparison of Calculated Trajectories, for Several Tests, With Corresponding Experimental Curves	16
6. Comparison of Calculated and Experimental Axial Temperature Distributions	18

## Illustrations

7. Plot of Lift-Off Point,  $x_c$ , Against Parameter,  $p$ , With  $m$  and  $\Delta T_o$   
as Parameters

20

# A Method for Determining the Point of Lift-Off and Modified Trajectory of a Ground-Based Heated Turbulent Planar Jet in a Co-Flowing Wind

## I. INTRODUCTION

As part of an investigation to aid in the development of an operational Warm Fog Dispersal System (WFDS), experimental and theoretical studies have been made of the characteristics of ground based heated jets with varying combinations of heat and thrust under different wind conditions. The numerical results obtained from these investigations will be utilized to help develop the control system for the WFDS. As part of the theoretical study, Klein and Kunkel<sup>1, 2</sup> described methods for calculating the characteristics of planar and round jets in a co-flowing wind; that is, in the same direction as the jet. The characteristics of heated jets in a counter-flowing air stream will be present in subsequent reports.

In the development of the previous models, the effect of the ground was neglected and the buoyant thrust of the air upon the jet was assumed to start acting as soon as the heated gas exited from the nozzle. The calculated trajectories consequently start to rise above the ground a short distance from the jet nozzle. Field

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(Received for publication 1 February 1977)

1. Klein, M.M., and Kunkel, B.A. (1975) Interaction of a Buoyant Turbulent Planar Jet With a Co-Flowing Wind, AFCRL-TR-75-0368.
2. Klein, M.M., and Kunkel, B.A. (1975) Interaction of a Buoyant Turbulent Round Jet With a Co-Flowing Wind, AFCRL-TR-75-0581.

results described by Kunkel<sup>3</sup> show, however, that the jet trajectories generally hug the ground for a considerable distance before the effect due to buoyancy is exhibited. This ground effect appears to stem from the development of a regime of recirculating flow in which the pressure is less than that of the surrounding air, Conway.<sup>4</sup> A complete description of the original model and its derivation for the planar case is contained in Klein and Kunkel.<sup>1</sup> The present report extends this model to take account of the ground effect.

Although desirable to be able to calculate the point of lift-off from first principles, it was felt that such an approach would be lengthy and complex. Further, in view of the incomplete knowledge of the details of the flow, the solutions obtained might not be accurate enough for utilization in the design of a control system for the WFDS. It was, therefore, decided to analyze the trajectory data available from the field experiments to see if the point of lift-off could be determined from the physical parameters defining the flow. Since Kunkel<sup>3</sup> found that the jets merge at about 2.5 times the combustor spacing, that is, a short distance down stream, the analysis was made for the planar jet case.

An examination of the data shows that, as anticipated, the point of lift-off is delayed by increasing the velocity of the jet and hastened by increasing the jet temperature. An obvious possible parameter would be the ambient Froude number, that is, the ratio of inertial to convective forces in the initial jet. However, the lift-off point must also depend upon windspeed and should be intimately related to the temperature and velocity at this location. The analysis was, therefore, made in terms of the local Froude number at the point of lift-off.

## 2. ANALYSIS

The point of lift-off,  $x_c$ , axial velocity,  $u_m$ , and axial jet temperature excess over ambient,  $\Delta T_m$  are indicated schematically in Figure 1. The local Froude number,  $F_1$ , at the point of lift-off, may be written as,

$$F_1 = \frac{u_m^2 T_a}{g b_o \Delta T_m} \quad (1)$$

3. Kunkel, B.A. (1975) Heat and Thrust Requirements of a Thermal Fog Dispersal System, AFCRL-TR-75-0472.

4. Conway, Arthur (Editor) (1975) A Guide to Fluidics, MacDonald & Co. (Publishers, Ltd), London, pp. 2-7.

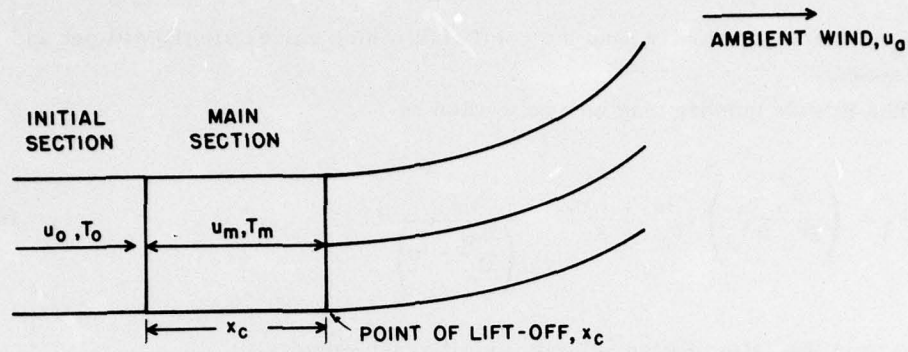


Figure 1. Schematic Representation of Jet and Lift-Off Point

where  $T_a$  is the ambient temperature,  $g$  the gravitational constant, and  $b_o$  the initial jet width. The functional relation  $f$ , if it exists, between  $x_c$  and  $F_1$  may then be expressed as

$$x_c = f(F_1) \quad (2)$$

It is convenient to write  $F_1$  in the form (the subscript zero refers to initial jet values)

$$F_1 = \frac{u_o^2}{\Delta T_o} \frac{u_m^2}{u_o^2} \frac{T_a}{g b_o \frac{\Delta T_m}{\Delta T_o}} \quad (3)$$

and express  $\Delta T_m$  in terms of the velocity increment  $z$  by (see Kunkel and Klein<sup>1</sup> for more detail)

$$\frac{\Delta T_m}{\Delta T_o} = kz \quad (4)$$

where

$$z = \frac{u_m - u_a}{u_o - u_a} \quad (5)$$

and  $u_a$  is the wind velocity, and  $k$  a coefficient which varies slowly with jet and wind speeds.

The Froude number may now be written as

$$F_1 = \left( \frac{u_o^2 T_a}{g b_o \Delta T_o} \right) \frac{u_m}{u_o} \frac{1-m}{k} \frac{u_m}{u_o \left( \frac{u_m}{u_o} - m \right)} \quad (6)$$

where  $m$  is the ratio of wind velocity to initial jet velocity.

For purposes of plotting, it is convenient to introduce a new number  $G_1$  in which the constant and nearly constant terms have been omitted, that is,

$$G_1 = \frac{u_o^2}{b_o \Delta T_o} \frac{u_m}{u_o} \frac{u_m(1-m)}{u_o \left( \frac{u_m}{u_o} - m \right)} \quad (7)$$

To see if  $G_1$  is a proper parameter for correlation of the data, a plot was made of  $x_c$  against  $G_1$  and a smooth curve, estimated by eye, drawn through the points (Figure 2). An examination of Figure 2 shows that, while most of the points lie close to the curve, several of the points — identified by tick marks — lie distinctly below. Since these points occur at the higher values of  $m$  ( $m \sim 0.1$ ), it appears that, as the windspeed increases, the point of lift-off decreases. This behavior, which is opposite to the expected increase of  $x_c$  with windspeed, may be explained in terms of the regime of low pressure recirculating air described by Conway. The decrease in pressure of the recirculating air will be determined, to a large extent, by the increase in velocity of the air mixing with the jet. As the windspeed increases, a smaller velocity increase is obtained, resulting in a weaker pressure decrease and earlier lift-off.

To take into account this behavior, the Froude number  $F_1$  in Eq. (1) was modified to

$$F = \frac{u_m(u_m - u_a)T_o}{g b_o \Delta T_o} \quad (8)$$

resulting in, as the modified form of Eq. (6)

$$F = \left( \frac{u_o^2 T_a}{g b_o \Delta T_o} \right) \frac{u_m}{u_o} \frac{(1-m)}{k} \quad (9)$$

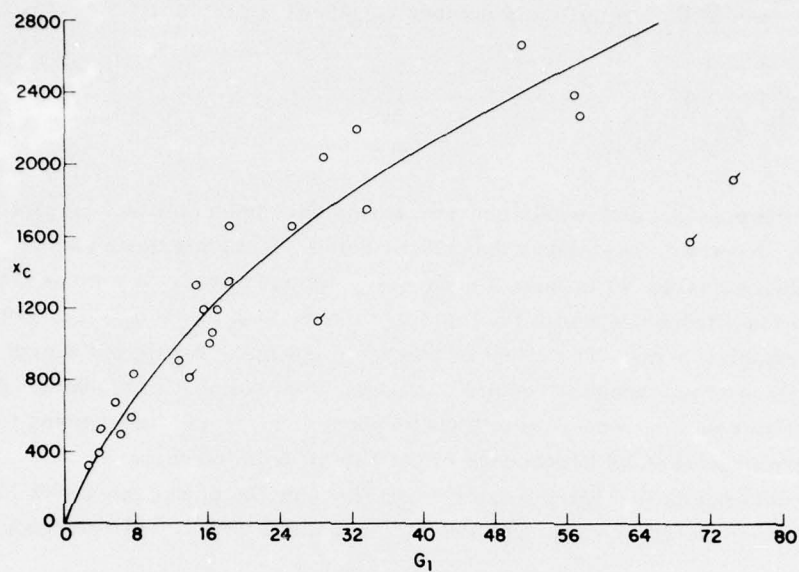


Figure 2. Plot of Lift-Off Point  $x_c$  Against Parameter  $G_1$  for Co-Flowing Test Data. Points with tick marks represent higher values of  $m$  ( $m \sim 0.1$ )

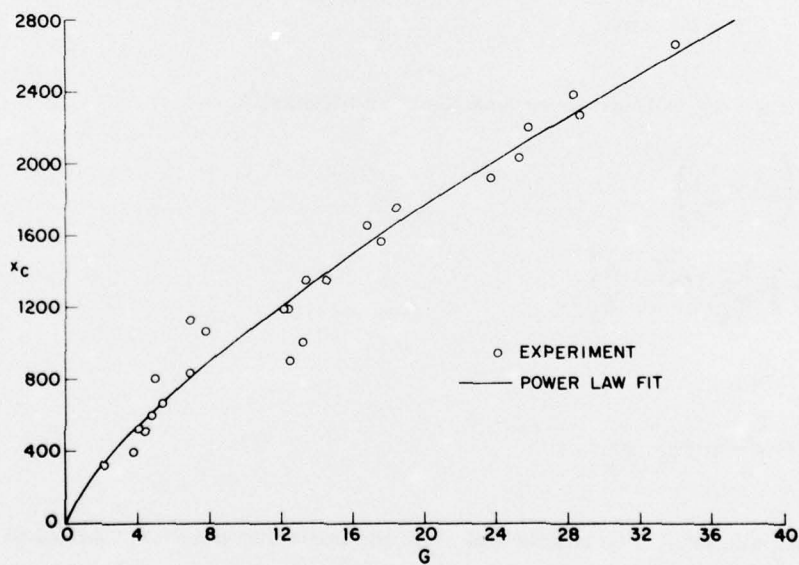


Figure 3. Plot of Lift-Off Point  $x_c$  Against Parameter  $G$  for Co-Flowing Test Data. Solid curve is power law fit

As in the case of  $F_1$ , we utilize a number  $G$  where

$$G = \frac{u_o^2 (1 - m)}{b_o \Delta T_o} \frac{u_m}{u_o} \quad (10)$$

in which the constant and nearly constant terms have been omitted. A plot of  $x_c$  against  $G$ , Figure 3, now shows that all the points, including those associated with the higher values of  $m$  form a reasonably defined curve. It may be noted that if we had modified Eq. (1) with the term  $u_o z$  rather than  $(u_m - u_a)$ , Eq. (10) would lack the factor  $(1 - m)$ . The effect of this term is almost negligible except at the higher values of  $m$ , where it results in a slight improvement of the data. Additional reliable data at moderate to higher values of  $m$ , would be required to determine more precisely the dependence of the lift-off point upon  $m$ .

The data points in Figure 3 can be well fitted by the power law curve (for purposes of correlation, distances are dimensionless with  $b_o$  the reference length)

$$x_c = 190.0 G^{0.744} \quad (11)$$

The velocity  $u_m$  is a function of  $x$  and may be calculated from  $z$  in Eq. (5) by

$$\frac{u_m}{u_o} = z(1 - m) + m \quad (12)$$

where, at  $x = x_c$ ,  $z$  is given by (see Klein and Kunkel<sup>1</sup>)

$$\frac{z}{a} = \left( \alpha^2 + \frac{x_o}{x_c} \right)^{1/2} - \alpha \quad (13)$$

$$\alpha^2 = \frac{1}{4} \frac{T_o}{T_a} \frac{m^2}{1 - m} \frac{A_1^2}{A_2} \quad (14)$$

$$a^2 = \frac{T_o}{T_a} \frac{1}{1 - m} \frac{1}{A_2} \quad (15)$$

$A_1$  and  $A_2$  are profile constants and  $x_o$  is the initial jet position. Although Eq. (11) can be solved numerically for  $x_c$ , a formal solution is not available because of the nonintegral exponent occurring in  $G$ . However, an explicit solution can be obtained if the variables  $x_c$  and  $G$  are changed to  $q$  and  $p$ , where

$$q = \frac{x_c}{u_m/u_o} \quad (16)$$

and

$$p = \frac{u_o^2(1-m)}{\Delta T_o b_o} \quad (17)$$

Note that the parameter  $p$  is independent of  $x_c$  and can be immediately calculated for a given set of flow conditions. Since the exponent of  $G$  in Eq. (11) does not differ greatly from unity, we may expect good correlation in a plot of  $q$  against  $p$ . This anticipated result is confirmed in Figure 4 where the scatter of the plotted points is seen to be small. A good fit to the data is provided by the power law curve

$$q = 253.64 p^{0.80} \quad (18)$$

with a correlation coefficient of 0.98.

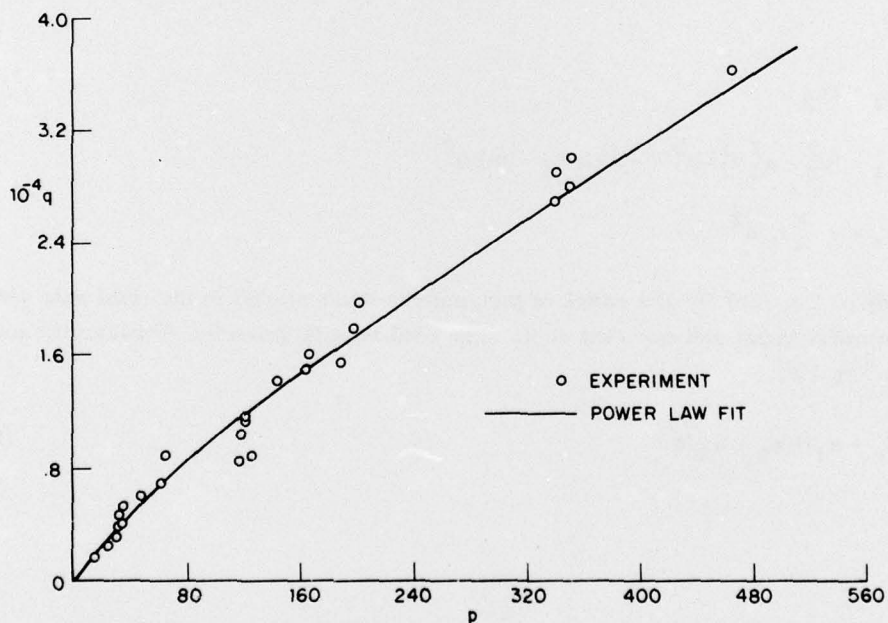


Figure 4. Plot of Parameter  $q$  Against Parameter  $p$  for Co-Flowing Test Data. Solid curve is power law fit

To obtain the lift-off point  $x_c$  for a given value of the parameter  $q$ , we utilize Eqs. (12) and (13) to express  $q$  in Eq. (16) as a function of  $x_c$ , yielding

$$q = \frac{x_c}{c_1 \left( \alpha^2 + \frac{x_0}{x_c} \right)^{1/2} - c_2} \quad (19)$$

where

$$c_1 = (1 - m) a$$

and

$$c_2 = \alpha c_1 - m.$$

Rationalization of Eq. (19) leads to the cubic

$$x_c^3 + a_2 x_c^2 + a_1 x_c + a_0 = 0 \quad (20)$$

where

$$a_2 = 2c_2 q$$

$$a_1 = (c_2^2 - \alpha^2 c_1^2) q^2 = -m(2\alpha c_1 - m) q^2$$

$$a_0 = -c_1^2 x_0 q^2.$$

A study of Eq. (20) for the range of parameters encountered in the field data yields two complex roots and one real root. The real root is given by Abramovitz and Stegun<sup>5</sup> (p 17),

$$x_c = s_1 + s_2 - a_2/3, \quad (21)$$

5. Abramovitz, M., and Stegun, I. (Editors), (1964) Handbook of Mathematical Functions, U. S. Government Printing Office, Washington, D. C.

where

$$s_1 = \left[ r + (t^3 + r^2)^{1/2} \right]^{1/3}$$

$$s_2 = \left[ r - (t^3 + r^2)^{1/2} \right]^{1/3}$$

$$r = \frac{q^2}{6} \left[ 2m(m - \alpha c_1)(2\alpha c_1 - m)q + 3x_o c_1^2 \right] + \frac{8}{27} (m - \alpha c_1)^3 q^3$$

$$t = -\frac{q^2}{27} \left[ 4\alpha^2 c_1^2 - m(2\alpha c_1 - m) \right] .$$

Since the jet characteristics at a given location are independent of the vertical position,  $y$ , the method of calculation presented in Klein and Kunkel<sup>1</sup> requires only minor modifications to take account of the ground effect. The buoyant force is taken as zero up to the lift-off point  $x_o$ , at which point it is assumed to start acting. The jet characteristics at this point are then calculated and utilized as the initial conditions for the subsequent buoyant motion of the jet. Of interest for fog dispersal work, in addition to the jet trajectory, is the temperature distribution along the jet,  $\Delta T_m$ , which is easily obtained from Eqs. (4) and (13).

### 3. RESULTS AND DISCUSSION

The calculated trajectories with and without ground effect and the corresponding experimental curves are presented in Figure 5. These results cover most of the range of initial jet speed, initial jet temperature, and ambient wind encountered in the field tests. The calculated trajectories are in fair-to-good agreement with the experimental curves, generally tending to be somewhat higher, especially at very low windspeeds. In addition, several of the calculated lift-off points are not in good agreement with the experimental values, a result due to the fact that several of the experimental lift-off points did not lie close to the correlation curve of  $q$  vs.  $p$  (Figure 4). This behavior in the lift-off point may be due to an instability of the air near the ground with respect to fluctuations in the ambient wind, under certain flow conditions. In test 16-9, the experimental curve shows a flattening tendency at large jet distances not exhibited by the calculated trajectory. An examination of the upper level wind read by the wind sensor, shows that it is almost double the ambient wind. A correction to the later portion of the jet trajectory, utilizing the

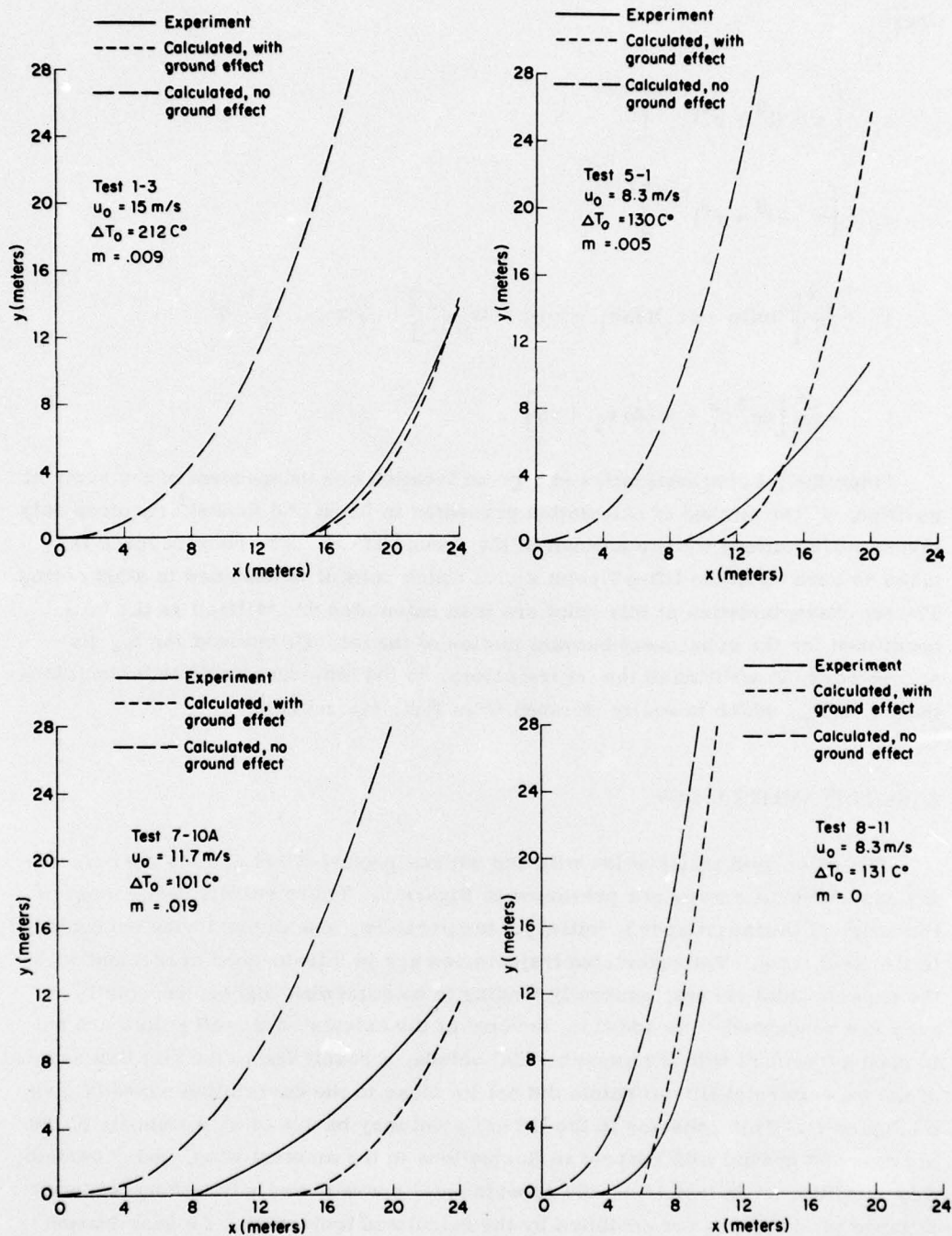


Figure 5. Comparison of Calculated Trajectories, for Several Tests, With Corresponding Experimental Curves

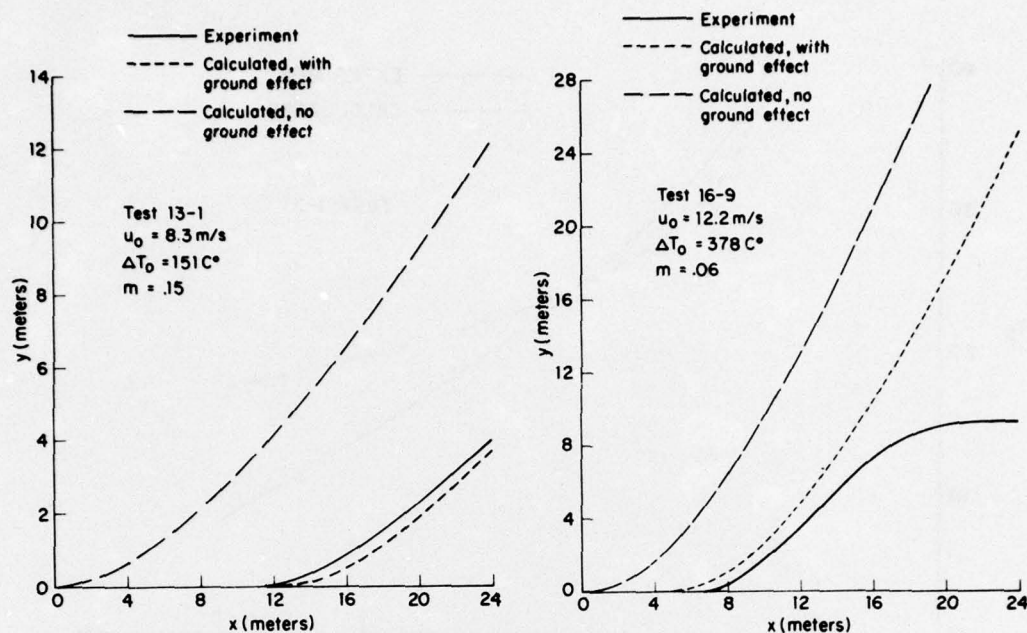


Figure 5. Comparison of Calculated Trajectories, for Several Tests, With Corresponding Experimental Curves (Cont)

proper jet influenced upper level wind, brings it into good agreement with the experimental curve. This flattening out of the trajectory is exhibited principally by those tests in which the ambient wind was strong enough to keep the trajectory from becoming very steep.

Typical calculated and experimental temperature distributions along the axis are presented in Figure 6 for two initial jet temperatures. In general, the calculated curves decrease at a somewhat slower rate than those obtained experimentally. This results in temperatures that are too high at large jet distances, with the effect being greater for the case of higher initial jet temperature. Since the jet is strongly inclined to the vertical at large jet distances, the value of the mixing coefficient  $c$  — valid for a co-flowing nonbuoyant jet — may be inaccurate in this region. In addition, the physical trajectory will be deformed by the dynamic pressure exerted by the ambient wind upon the jet when the angle between the two flows is large.<sup>6</sup> A more accurate model, incorporating these and other possible modifications, is beyond the scope of the present investigation.

6. Abramovich, G. N. (1963) The Theory of Turbulent Jets, The MIT Press, Cambridge, Mass., pp. 541-556.

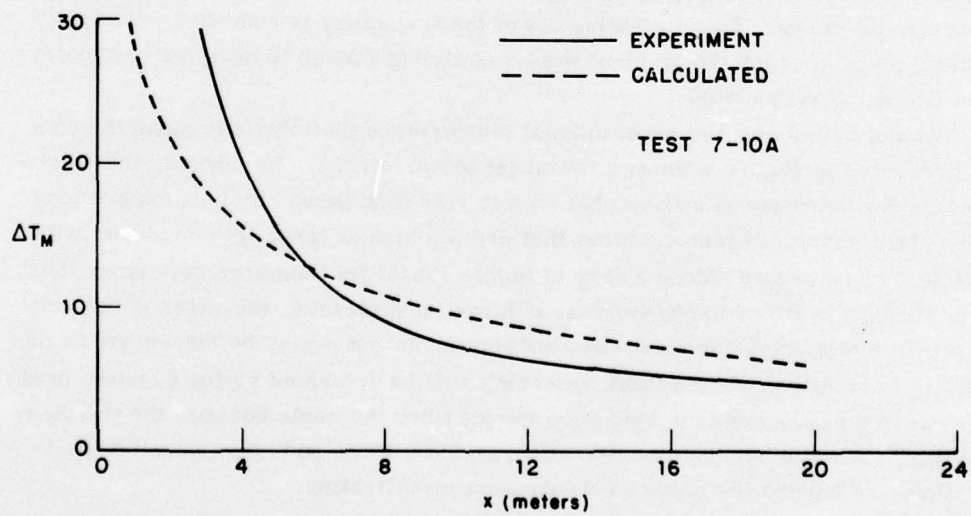
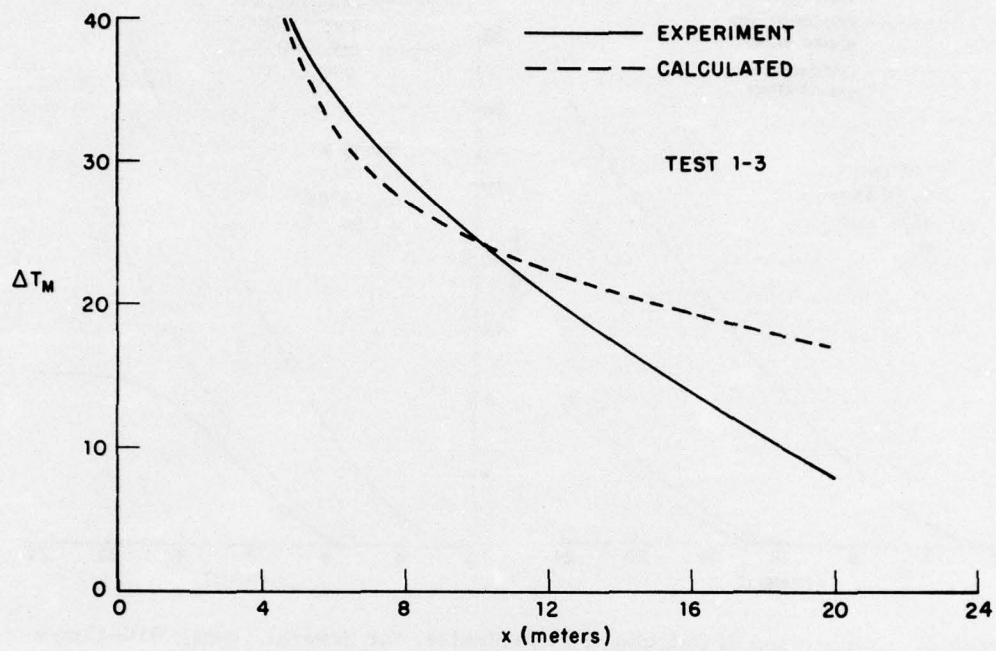


Figure 6. Comparison of Calculated and Experimental Axial Temperature Distributions

#### 4. PROCEDURE FOR RAPID DETERMINATION OF LIFT-OFF POINT

For use in field work, it is desirable to be able to rapidly estimate the lift-off point of the jet. An examination of Section 2 shows that  $x_c$  depends explicitly upon  $m$  and  $\Delta T_o$  and implicitly upon  $b_o$  and  $u_o$  through the parameter  $p$ . The solution for  $x_c$  may, therefore, be presented in fairly compact form without explicit reference to  $b_o$  and  $u_o$  provided  $p$  is used as the independent variable. Calculations of  $x_c$  have accordingly been made for a range of input data, and plotted in Figure 7 against  $p$ , with  $m$  and  $\Delta T_o$  as parameters. As seen from the figure, the lift-off point is much more sensitive to  $m$  than to  $\Delta T_o$ , the weak dependence upon the latter becoming very small when  $m$  is increased to 0.2.

For a given set of flow conditions, the value of  $x_c$  is easily obtained from Figure 7. As an illustrative example we take:

$$u_o = 16 \text{ m/sec} , \quad u_a = 0.9 \text{ m/sec}$$

$$T_o = 175 \text{ K}^{\circ} , \quad b_o = 0.01 \text{ m} .$$

The wind parameter  $m$  is given by

$$m = \frac{u_a}{u_o} = \frac{0.9}{16} = 0.056 .$$

The parameter  $p$  may now be calculated from

$$p = \frac{u_o^2 (1 - m)}{b_o \Delta T_o}$$

to yield

$$p = \frac{256(0.944)}{0.01(175)}$$

$$= 138.1 .$$

From Figure 7, by interpolation

$$x_c = 1520, \text{ in units of } b_o$$

$$= 15.2 \text{ m} .$$

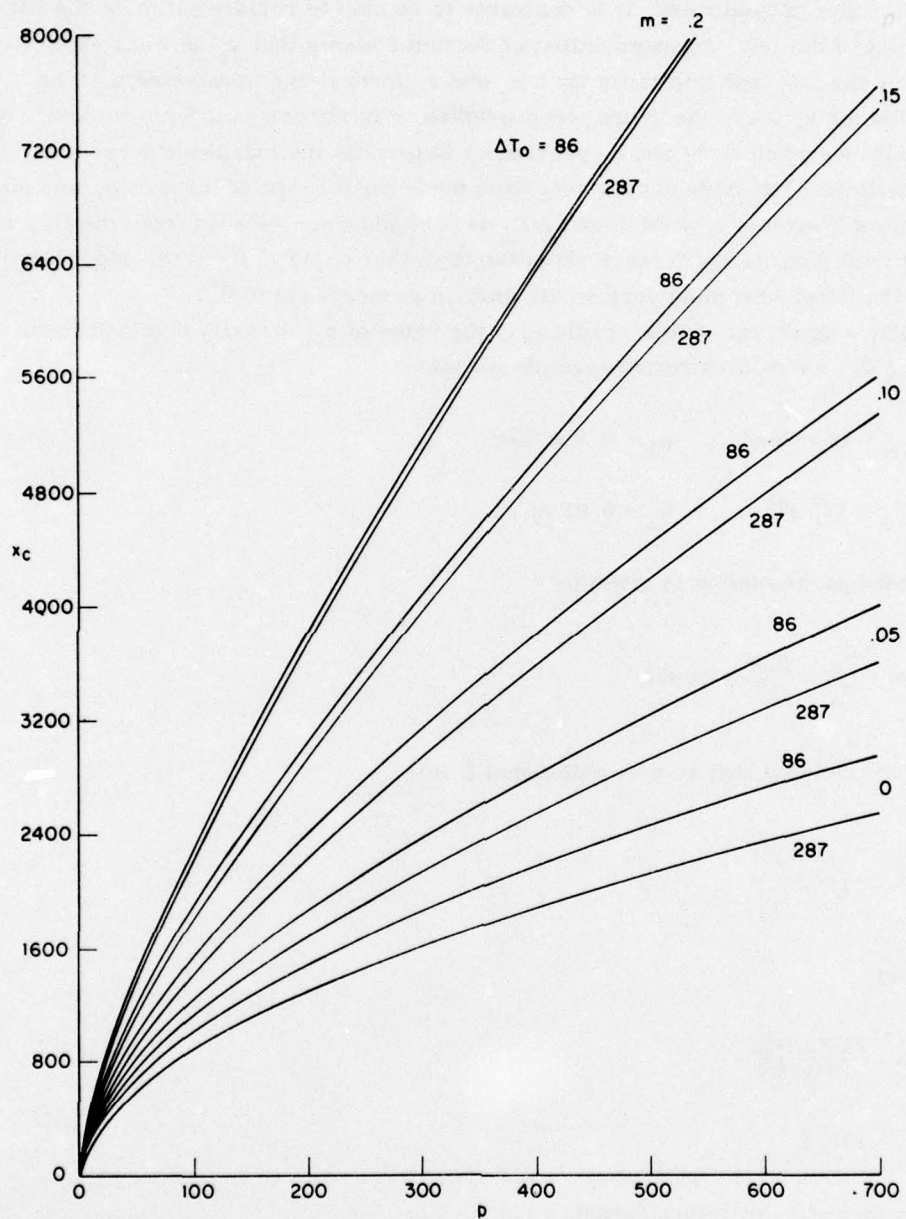


Figure 7. Plot of Lift-Off Point,  $x_c$ , Against Parameter,  $p$ , With  $m$  and  $\Delta T_0$  as Parameters

## 5. SUMMARY AND CONCLUSIONS

The method of calculating the dynamic characteristics of a ground based turbulent heated planar jet has been extended to take account of the tendency of the jet to remain attached to the ground for a considerable part of the jet trajectory. The lift-off points, determined from the experimental data, have been analyzed and correlated in terms of the local Froude number at the lift-off point. From this analysis, a procedure has been developed for determining the lift-off point when the flow conditions for the jet — that is, ambient wind, initial jet velocity, and temperature — are given. The new jet trajectory may now be calculated, with only a minor modification of the original procedure. The calculated trajectories, and temperature distributions along the jet are in fair-to-good agreement with the corresponding results from the field data. The present method of calculating the jet trajectories appears accurate enough to be utilized in the development of a control model for the fog dispersal system.

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